FORESTRY

FOREST RESOURCE DEVELOPMENT AGREEM

Canada

BC

FRDA REPORT 071

THE SOCIAL DISCOUNT RATE FOR SILVICULTURAL INVESTMENTS

by

Terry Heaps and Brian Pratt

Contractor: Terry Heaps
Department of Economics
Simon Fraser University
Burnaby, B.C.
V5A 1S6

Prepared for: Ministry of Forests
Industry Development and Marketing Branch
Victoria, B.C.

and

Forestry Canada
Pacific and Yukon Region
Economic Analysis Section
Victoria, B.C.

March 1989

This report has been reviewed by the British Columbia Ministry of Forests and Forestry Canada and approved for distribution. Approval does not necessarily signify that the contents reflect the views and policies of the British Columbia Ministry of Forests or Forestry Canada. Mention of trade names or commercial products does not constitute recommendation or endorsement for use.

ISSN 0835-0752

89-05056/

Canadian Cataloguing in Publication Data

Heaps, Terry.
The social discount rate for silvicultural investments

(FRDA report, ISSN 0835-0752; 071)

On cover: Forest Resource Development Agreement

Bibliography: p.
ISBN 0-662-16985-9
Fo29-19/071E

1. Forests and forestry - Canada - Finance. 2.Public investments - Canada - Cost effectiveness. 3. Rate of return - Canada. I. Pratt, Brian. II British Columbia. Ministry of Forests. Industry Development and Marketing Branch III. Canada. Forestry Canada. Pacific and Yukon Region. Economic Analysis Section. IV. Canada- British Columbia Forest Resource Development Agreement. V. Title. VI. Series.

SD393.H42 1989 338.1'3495'0971 C89-092123-7

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ABSTRACT

The method by which the real social discount rate has been estimated for Canada is reviewed. Some of the assumptions which were used to obtain the current rate of 8 - 10% are criticized. In particular, it is shown that that the real marginal and average rates of return on investment are not equal. A method of estimating the real marginal rate of return on investment is found. This method and other information is used to revise the estimate of the real social discount rate to 3 - 7%. It is proposed that where possible expected net present values should be used to screen public investments in silviculture. An argument is made that the benefits and costs of such investments should not be adjusted for risk. The risk free social discount rate should be used for expected net present value calculations. Historically, this rate has ranged between 3 and 5%.

Executive Summary

Current government guidelines for conducting cost benefit analysis of public sector investment proposals state that projects should be rejected if analysis yields a negative net present value. The guidelines recommend that a real social discount rate of 8 to 10% be used in the calculation of the net present value (Loose, 1977). Most proposals to invest in silvicultural projects would be rejected under these guidelines (Heaps, 1985). Therefore, this paper critically reviews the reasoning that was used to justify a discount rate of 8 - 10%.

The analysis presented in this paper shows that the use of the current guidelines will lead to a systematic undervaluation of the net social benefits of long duration investments in forestry. The major results are summarized below.

(a) Any empirical estimate of a real social discount rate should be adjusted downwards to account for the observed difference between average and marginal tax rates.

(b) The discount rates recommended in the current government guidelines should not be used to screen public sector investment proposals because:

(i) they have not been adjusted to account for differences between average and marginal tax rates and hence;

(ii) they are not estimates of the (theoretically relevant) marginal social discount rate.

(iii) they overstate the social opportunity costs of public sector investments because they are based on erroneous estimates of the returns from private sector investments.

(iv) they contain a risk premium which does not reflect the social risks of investments by the public sector, and investments in silviculture in particular.

(c) The marginal, risk adjusted, social discount rate for Canada does not exceed 7% for the 1965 - 1974 period (which is the period of estimation for the 8 - 10% rates).

(d) The discount rate in (c) above can be updated in a theoretically consistent way by using the extremely rich sources of data on rates of return in financial markets.

(e) Current estimates of the real yields on government bonds should be used as a discount rate when screening investment proposals in silviculture (historically this rate has been between 3 and 5%).

(f) The existence of risk and uncertainty should not be used as a rationale for undervaluing the net economic benefits of silvicultural projects by using a risk adjusted discount rate.

1. Introduction*

This study will critically examine some of the procedures that have been put forward for doing cost-benefit analysis of public projects. Particular attention will be paid to the choice of a discount rate for net present value (NPV) calculations. The emphasis will also be on projects such as investments in silviculture where the return to the investment will not be realized for a substantial number of years.

Current government guidelines, both for British Columbia and Canada, recommend that a real discount rate of the order of 8 - 10% be used in weighting future benefits and costs (Loose, 1977). In the case of most silvicultural projects, the use of a 8% discount rate would result in these projects having negative net present values (Heaps, 1985). Thus it is important, in the context of forestry, that the arguments that led to the 8 - 10% real discount rate be reviewed critically.

Such a review is carried out in this paper. It will be argued that several of the assumptions made in justifying a 8 - 10% social discount rate are not valid. The assumption that we criticize most strongly is the equality of the marginal and average social rates of return on private investment. We then modify the assumptions and conclude that the social discount rate should be in the range 3 - 7%. This range is consistent with the calculations of several other critics of the 8 - 10% rate. Finally, we argue that investment in silviculture is not very risky provided that managers are flexibile in making decisions about when to harvest the additional wood. Thus the risk free social discount rate can be used for silvicultural projects and this is in the range 3 - 5%.

A review of the determination of the social discount rate requires an understanding of the purpose of a NPV calculation. This purpose is to determine if a public project will generate higher returns for society than the alternative of allowing the private sector to use the funds for additional consumption and private investment in the way it sees best. The public project alternative yields the higher economic return when its NPV is greater than 0. NPVs can also be used to determine which of several projects yields the highest returns. The role of the discount rate in the calculation of NPVs is to provide a measure of the annual social returns to private sector wealth (at the margin). As such it is a weighted average of the return

* We are grateful for comments received from Bill Howard of the B. C. Ministry of Forests and Bill White of Forestry Canada, Pacific Forestry Center. Any errors in this paper, however, are our responsibility alone.

needed to induce society to forego current consumption (i.e., the consumption rate of interest), the return needed to induce additional private investment (i.e., the marginal rate of return to private investment) and the social cost of foreign borrowing.

NPV calculations take into account the compounding of private sector wealth that occurs as time passes. Thus if the annual social return on private sector wealth was 10%, then $1 held today would be worth $6.73 after 20 years ($6.73 = (1.10)^{20}$). However, if this rate of return was only 5%, then the $1 would be worth only $2.65 after 20 years. These examples of the effect of compounding show that a correct determination of the discount rate is of critical importance in the evaluation of investments in silviculture. For example, a fertilization of a stand intended for harvest 20 years from now would have to yield $6.73 in additional net revenues per $1 of treatment costs in order to have its NPV greater than 0 if the correct discount rate was 10%. This is 150% more than would be needed to pass this same test if the discount rate was 5%

The theoretical rationale for using net present value calculations to screen investment proposals was Irving Fisher's model of the consumption rate of interest. In his model of a perfectly competitive economy with no transactions costs, no taxes, no restrictions on credit and perfect information, it can be shown that the consumption rate of interest i equals the marginal real rate of return on investment r. Moreover, real investment activity will increase the real wealth of the society if and only if the net present value of that investment, using i = r as the discount rate, is positive.

However, when economic conditions depart from those of Fisher's "ideal" economy such as happens when there is unemployment, imperfect competition, taxes or risk and uncertainty, then the consumption rate of interest no longer equals the marginal real rate of return to private investment. Most of the research on capital expenditure analysis in the last 80 years has focussed on finding a "second best" discount rate which can be used to screen investment proposals in this imperfect world.

The methodology used to estimate a discount rate for public investments in Canada was developed by Harberger (1972). He developed a model of the market for loanable funds in which which the social discount rate can be identified with a weighted average of the marginal before corporation tax real rate of return on private investment and the after income tax real rate of return on private savings. Formulas are given for the weights in terms of the elasticities of the investment demand curve and the savings supply curve. Since foreign borrowing is also important in

Canada as a source of private and public investment funds, the social cost of foreign borrowing must also be included in defining the weighted average appropriate for Canada.

Jenkins (1973, 1977) applied Harberger's procedure to Canadian data and it was his calculations that led to the adoption of a 10% social discount rate by Treasury Boards in Canada. However, Jenkins' calculations are suspect on a number of grounds. First, there was little theoretical rationale for the way the weights were chosen. He asserted that all the relevant elasticities were equal to one. Secondly, the calculations of sectoral rates of return are based on accounting data and vary considerably from rates of return based on financial market data. Further, these calculations produce average rates of return rather than marginal rates of return and they do not deal with risk in an appropriate way. Consequently, Jenkins' figures have been severely criticized in the literature (Summer, (1980, 1980a), Burgess, (1981), Spiro, (1987)).

Section 2 of this study looks at what NPV calculations are intended to do. The social opportunity costs of raising the project funds are discussed in some detail and the discussion leads to Harberger's formula for the social discount rate. The potential problems with Jenkins' calculations are then discussed in more detail. The question of whether marginal and average social rates of return on private investment can be equal is examined. An intuitive explanation is given of why these rates may diverge. This explanation is based on the operation of the Canadian tax system.

The issue of divergence must, however, ultimately be addressed empirically. This is done in Section 3 where a model of the investment behaviour of firms, developed in Appendix 1, is used to provide a theoretically consistent methodology for obtaining estimates of marginal rates of return. This model is applied as much as is possible to Jenkins' data to show that there should be a significant downwards adjustment of Jenkins' estimate of the social discount rate. Indeed, the social rate of discount on this interpretation should be no more than 8%. This section also shows that there is a severe discrepancy between the real rates of return on financial assets and the rates of return calculated by Jenkins.

The next section reviews other literature which has criticized Jenkins' estimate of the social discount rate. Burgess (1981) argues that the elasticities used by Jenkins were incorrect. Burgess produces an estimate of the social discount rate of 7% when his modifications of Jenkins' elasticities are used in Jenkins' formulas. We also comment on Boadway et. al. (1984) who have developed a method of estimating the social discount rate which is based on the

rates of return in financial markets and various taxation parameters. Their procedure is based on a multiperiod extension of the model of Appendix 1. Using this method, they obtain an estimate of 6.5% for the social discount rate for the period 1963 -1971. Spiro (1987) has modified the Boadway et. al. calculations and obtained an estimate of the social discount rate of 6.6%. We make some adjustments to these calculations to come up with an estimate of 6.06%. In sum, all these studies except Jenkins seem to agree that the social discount rate should be 7% or less. These studies also contain estimates of the the consumption rate of interest of which the lowest is 3% so this figure can be taken to be a lower bound on the social discount rate.

The fifth section of this paper then deals with the treatment of risk and uncertainty in cost benefit calculations. The first point made is that where probabilities can reasonably be assigned to the possible outcomes of an investment proposal, then these probabilities should be used to calculate the expected net present value of the proposal. This is the figure that should be used to screen the proposal. Two examples of such calculations are given in Appendix 4.

The second point made is that the use of a single risk adjusted discount rate cannot be justified in these calculations (Myers and Turnbull (1977)). Thus the risk free rate of discount should be used but possibly some side adjustment should be made to the expected net present value in order to take account of the risks involved. This risk adjustment should be based on the contribution of the proposal to the risk that people in the economy face on their total portfolio of assets (Wilson (1982)). We argue that as long as there is flexibility in project scheduling in the sense that the manager can choose to collect returns from the project mainly when economic conditions are favourable, then this risk offset can be taken to be zero. It should also be noted here that a popular view in finance today is that if such options are available to the manager, then they may actually have a positive value and the expected net present value may well understate the true value of the proposal (Brennan and Schwartz (1985)). Finally, we recommend the use of sensitivity analysis when faced with events for which probabilities cannot reasonably be assigned (i.e., such as for future price uncertainity).

This study then concludes with a section of recommendations. As mentioned above, the final recommendations are that a discount rate of 3 - 5% be used in evaluating long lived silvicultural investments. The lower range of rates discussed above is chosen because it is believed that a 7% rate still contains a significant risk premium. Also where possible the expected net present value methodology should be used.

2. The Theory of Rates of Return

This section describes the approach used by Harberger (1972) and Jenkins (1977) to define and estimate the social discount rate. The meaning of the social discount rate is explained and then the market for loanable funds is discussed in some detail in order to justify Harberger's formula. We then point out that some of the assumptions made by Jenkins in applying this formula to Canada can be questioned. An intuitive explanation is given of why one of his assumptions may not be warranted. This assumption is the statement that the social marginal rate of return to private investment equals the social average rate of return to private investment The explanation is based on a theoretical model of investment demand under the Canadian tax system which is developed in Appendix 1.

We begin by noting that a public project must be financed either by government borrowing (domestic and foreign) or by the government imposing additional taxes. Our object is to determine if the net economic benefits generated by the public project exceed the net economic benefits that would accrue to private individuals and government if the money borrowed was left in private hands or if the additional tax revenues were used not on the project but instead to retire government debt.

Consider now a simple example of a public project where C is spent today and B is earned one year from now. The social rate of discount can then be defined as the number r_S such that $r_S C$ is the net social benefit of putting or leaving the funds C in private hands. The public project yields a higher return to society than the private alternative provided $B - C > r_S C$. It is easily seen that this relationship is equivalent to the project net present value NPV being positive. That is:

$$NPV = B/(1 + r_S) - C > 0,$$

This result in also valid for projects where the benefits and costs are spread over a number of years. The question is how should r_S be measured?

The determination of r_S is based on the economic theory of a competitive economy in which there are differences between private and social rates of return primarily because of taxation. Social benefits include all income earned by private individuals but private benefits include only income left over after taxes have been paid. We will suppose now that there is a market for loanable funds and thus a supply curve S(r) for domestic savings, a demand curve I(r) for private investment and a supply curve F(r) for foreign financial capital. Here r is the market rate of interest.

It can be interpreted as the lowest or marginal rate of return for those private investments that are carried out. Since those who supplied the funds for these investments must be paid this rate of return it is what has been earned on these investments after all corporation and sales taxes have been paid. The social marginal rate of return on these investments is thus r plus the taxes paid on the marginal investment adjusted for investment subsidies. The market rate of interest is also the income that lenders earn per dollar of their savings.

The market rate of interest can also be interpreted as the rate of return required by lenders in order for them to be induced to save the requisite amounts. It should be noted that interest earned on savings is subject to personal income tax in the case of domestic savers and withholding tax in the case of foreign lenders.

Suppose now that the market for loanable funds is in equilibrium. That is the demand for loanable funds equals the supply of loanable funds. If D is the government deficit or how much the government must borrow for the year then this equality can be written as

$$D + I(r) = S(r) + F(r)$$

Consider now the effect of increasing the deficit by C which is the difference in borrowing requirements between carrying out the public project or instead using the additional tax revenue required by the project to retire government debt. Then there will be a small increase in r leading to increases ΔS and ΔF in savings and foreign loans and a reduction $-\Delta I$ in private investment (where $\Delta I > 0$). For the market for loanable funds to remain in equilibrium

$$C = \Delta S + \Delta I + \Delta F$$

There are also social costs in the form of foregone private income associated with these changes. Consider first those who save the additional ΔS. They do not lose this income. They simply lose the opportunity to spend it in the current period. To continue with simple examples, we will suppose that they recover both principal and interest on their loan at the end of one year. They will have then an income of $\Delta S(1 + i)$ after they have paid personal income tax on the gross of tax interest, ΔSr, that they receive. The number i is r minus the marginal personal income tax rate. The increase in their income over one year must be the value to them of the lost opportunity for additional current consumption. Thus ΔSi is the social (opportunity) cost of saving an additional ΔS.

Next looking at investment, the fall in investment leads to less private income and less corporation tax paid. The loss will be $\Delta I\gamma$ where γ is the marginal gross of tax return on private investment so γ is r plus an adjustment taking into account corporation income tax net of any tax credits or subsidies.

Finally, government must repay foreign lenders the equivalent of an amount of $\Delta F(1 + \rho')$ after one period has passed. Here r' is the marginal rate of return required by foreign lenders in terms of Canadian dollars. The net loss of income to government and society is $\Delta F\sigma$ where σ is r' minus the marginal rate of withholding tax on interest payments to foreigners.

Now summing up, the total social (opportunity) cost of the increase in government borrowing , which was defined to be $r_S C$, is

$$r_S C = i\Delta S + \gamma\Delta I + \sigma\Delta F$$

Harberger (1972) and Jenkins (1977) allow in this equation for gross of tax marginal rates of return on private investment to be different across a number of industrial sectors and for i to vary across income classes (perhaps because of differences in rates of taxation). This is easily accomplished by breaking ΔS and ΔI into their components and multiplying each component by the appropriate rate of return.

The two equations above combine to give a formula for the social discount rate. This formula is

$$r_S = \frac{i\Delta S + \gamma\Delta I + \sigma\Delta F}{\Delta S + \Delta I + \Delta F}$$

Thus r_S is a weighted average of the three rates of return i, γ and σ where the weights sum to one. It is also possible to rewrite the above formula as

$$r_S = \frac{iS\varepsilon_S + \gamma I\varepsilon_I + \sigma F\varepsilon_F}{S\varepsilon_S + I\varepsilon_I + F\varepsilon_F}$$

In this formula

ε_S = the interest elasticity of the supply curve for domestic savings
ε_I = the negative of the interest elasticity of the investment demand curve
ε_F = the interest elasticity of the supply curve for foreign financial capital

The problem now is to estimate all the items in the above equation. Jenkins (1977) assumes that the magnitude of each of the three elasticities is one. Others have quarrelled with this assumption (see section 4). This assumption implies that any additional government borrowing will be financed from S, I and F in exactly the same proportions as all other government borrowing and thus denies the government any discretionary power in the choice of methods of financing new public programs. This assumption does make it easy however to estimate the weights in the above equation. The weight for savings is the ratio of savings to the government deficit and so on.

We are also very concerned about the way Jenkins estimated the marginal social rate of return on industrial investment. This number is important because it receives a weight of 59% in the calculation of r_S. Jenkins claims that the marginal social rate of return on investment is equal to average social rate of return on investment and then estimates the latter number. However, he has provided no theoretical or empirical justification for this assertion (for example, see Jenkins (1981)). Thus, we have developed a theoretical model which enables the comparison of the marginal social and average social rates of return on investment. The model leads to a method of estimating the marginal social rate of return on investment.

Appendix 1 contains a full description of this model. It is a simplified macroeconomic model of investment behaviour where firms are assumed to take into account the Canadian tax system. Thus there is a wedge between private (after corporation tax) rates of return and social (before tax) rates of return. There is also a wedge between the private and social costs of acquiring capital because of the federal sales tax and tax subsidies such as investment tax credits and the capital cost allowance.

It is assumed that firms invest in the various types of capital in a manner so as to maximize the wealth of their shareholders. This is shown to be equivalent to the private (after tax) marginal rate of return on each type of capital being equated to the market rate of interest, namely r.

The model provides a theoretically consistent way of estimating the marginal and average social rates of return to investment by gathering data on r, economic depreciation rates, private and social costs of purchasing capital and tax rates. We also at this point distinguish between the average tax rate t_A (total provincial sales tax and corporation income tax divided by the social cost of the capital stock) and marginal tax rates t_M^1 (taxes paid on the returns from the last unit of capital purchased divided by the social cost of that unit). Since t_A is related to the average profitability of capital and t_M^1 is related to the

marginal value product of the ith type of capital, there is no reason to suppose that these rates have similar magnitudes.

The expressions found for the marginal and average rates of return to investment (r_{MS} and r_{AS}) are quite complicated but in the case where the social and private costs of acquiring capital are identical they simplify to

$$r_{MS}^{i} = r + t_{M}^{i}$$

$$r_{AS} = r + (1 + r)NPV/VS_0 + t_A$$

The first equation says that the social marginal rate of return to investment is just the private marginal rate of return to investment plus the marginal tax rate. Similarly the second equation says that the social average rate of return to investment is the private average rate of return to investment plus the average tax rate. This is because it is shown that $r + (1 + r)NPV/VP_0$ is the private average rate of return to investment. The term $(1 + r)NPV/VP_0$ is the end of period value of any rents earned by the firm divided by the initial value of the firm which here equals VS_0.

For the purposes of estimation it will be assumed that the economy as a whole is sufficiently competitive so that the value of rents relative to the social value of the capital stock should be very small. Then letting r_{MS} and t_M be weighted averages of individual marginal rates of return and tax rates we see that a divergence between social marginal and average rates of return occurs when marginal and average tax rates differ. That is:

$$r_{MS} - r_{AS} = t_M - t_A$$

This last equation holds even when the private costs of acquiring capital differ from the social costs of acquiring capital. Jenkins (1977) has estimated r_{AS} and t_A. One more equation is needed to estimate r_{MS} and t_M. Above t_M^i was related to the marginal product of the i^{th} type of capital. The condition for profit maximization is that the net of corporation tax marginal value product of the i^{th} type of capital should be set equal to the user cost of capital. Thus the marginal tax rate can be related to the user cost of capital. This equation together with the equation above leads to an expression for r_{MS} which can be estimated using Jenkins' data (see the next section or Appendix 1 for the final result).

3. Revised Estimates of the Social Discount Rate

Appendix 1 develops a theoretical model of investment demand given the Canadian tax system. The purpose of the model is to provide a theoretically consistent method of estimating the marginal social rate of return on investment. This rate is important, as shown in the previous section, in Jenkins' estimation of the social discount rate. Here the estimating equation for r_{MS}, which is reproduced below, provides an answer to the statement in Jenkins (1981) that "there is scant evidence to suggest that the marginal expected return on new investment is significantly different from average observed rates of return". The variables in the equation below are: t the real average corporation income tax rate; t_s the retail sales tax rate; r_{AS} the average social rate of return on investment; t_A the taxes generated by investment divided by the amount of investment; d_K the average rate of appreciation of the replacement cost of capital net of physical depreciation.

$$r_{MS} = ((1 + t_s)(r_{AS} - t_A) - (t_s + t)d_K)/(1 - t) \qquad (29)$$

The data needed for these variables can, for the manufacturing sector, mostly be found in Jenkins (1977, 1985). The relevant numbers for this sector are contained in Table 1 below.

Table 1

Data for the Manufacturing Sector 1965 - 1974

Year	Average Private Rates of Return $r_{AS} - t_A$	Net Economic Depreciation d_K	Real Average Corporation Tax Rates t
1965	6.78%	1.45%	35.01%
1966	6.32	3.80	31.48
1967	5.39	7.09	36.31
1968	6.04	5.88	41.04
1969	6.39	3.79	40.33
1970	4.29	3.71	45.61
1971	5.19	3.04	39.85
1972	5.87	4.92	38.84
1973	7.00	4.10	32.18
1974	6.17	4.43	30.07
Average	5.94	4.22	37.07

Source: Jenkins (1977) Table 2-6, p.39; Table 2-7, p.43; Table D-1, p. 198; Table D-7, p. 212.
Jenkins (1985) Table I, p. 30.

The one additional variable needed is the retail sales tax rate. We use 6% for this rate (the median provincial rate given in Boadway and Kitchen (1984, Table 5-3)) for all years believing that this overstates the actual rates of sales tax for 1965 - 1974 and hence biases our estimates of r_{MS} upwards.

Our estimates of r_{MS} for the manufacturing sector are presented in Table 2 together with Jenkins estimates of the average social rate of return r_{AS}. The third column in this table gives the difference between these two figures. It appears that the marginal rate was 10.18% on average for 1965 - 1974 as compared to Jenkins' average rate which was on average 13.18% for this period.

Table 2

Estimated Real Social Rates of Return for the Manufacturing Sector 1965 - 1974

Year	Marginal Social Rates of Return r_{MS}	Average Social Rates of Return r_{AS}	The Difference $r_{AS} - r_{MS}$
1965	10.15%	14.01%	3.86%
1966	9.93	12.58	2.65
1967	10.10	11.96	1.86
1968	11.17	14.06	2.89
1969	10.86	14.25	3.39
1970	7.65	10.61	2.96
1971	8.57	12.09	3.52
1972	10.41	13.04	2.63
1973	11.67	15.08	3.41
1974	10.75	14.08	3.33
Average	10.13	13.18	3.05

Source: r_{MS} was calculated from (28) using the data in Table 1 and $t_s = .06$. r_{AS} is from Jenkins (1977) Table 2-7, p.43.

Jenkins' estimate of the social rate of return for the manufacturing sector then becomes a major component of his calculation of the social rate of return for the total industrial sector. This rate is calculated as a weighted average of rates for manufacturing, nonmanufacturing, mining and petroleum and the financial sector. Using the same methodology as for the manufacturing sector we calculated a marginal social real rate of return for nonmanufacturing of 8.94% whereas Jenkins' average real social rate of return

for this sector was 9.44% (see Appendix 2 for the details). We are unable to estimate marginal rates of return for the other industrial sectors, however, because the requisite data was not available in Jenkins. We therefore accept his average rates as also being the marginal rates for these sectors. Jenkins then computes an estimate of the average social real rate of return for the industrial sector as a weighted average of the sectoral average rates of return. The weights are the sectors shares of the total industrial capital stock (computed from tables 2-1, 2-7, 3-1, 3-2 and 3-5 in Jenkins (1977)). We use the same weights as Jenkins to compute our revised estimate of r_{MS} for the industrial sector. We also reject his adjustment for rents he claims are earned by labour on the grounds that these must be zero at the margin. In sum then, our revised estimate of the marginal social real rate of return for the industrial sector is 9.37% as compared to Jenkins' estimate of 12.53%.

Jenkins' final step in estimating the social discount rate is illustrated in the next table. As explained in section two, the social discount rate is a weighted average of the sectoral social opportunity costs. Jenkins makes simplifying assumptions so that the weights are the sectoral shares in financing the government deficit as given in the third column of Table 3.

Table 3

Estimates of the Social Discount Rate 1965 - 1974

Sector	Social Opportunity Costs		Sector Financing Contributions
	Jenkins	Revised	
Industrial	12.53%	9.37%	.59
Residential Construction	7.50	7.50	.16
Agriculture	4.48	4.48	.00
Domestic Consumption	4.14	4.14	.05
Foreign	6.11	6.11	.20

Source: The first and last columns are from Jenkins (1977) Table 6-2 , p. 137 and Table 6-3, p. 140. The second column incorporates our revision of social opportunity cost of capital for the industrial sector but accepts Jenkins' figures for the other sectors.

This procedure gives a social discount rate of 10.02% using all of Jenkins' figures and a revised estimate of 8.16% using our revision of the figure for the industrial sector. Hence our substitution of marginal rates for average rates (wherever possible) has resulted in a reduction of Jenkins' estimate of the social discount rate by 1.86% from 10.02% to 8.16%.

Apart from mistakenly taking average and marginal rates of return to be equal, we believe there are at least two other reasons to be suspicious of Jenkins' calculations. First, there is very little variation in his private rates of return from year to year. This is not consistent with the fact that there is a substantial amount of risk in manufacturing. Secondly, there is no correlation at all between his numbers and the actual realized rates of return on debt and equity claims against Canadian corporations. This lack of correlation is obvious when Jenkins' estimates of real average private rates of return are compared to the estimates of real average private rates of return on common stocks presented in Pesando(1983, Table 4) as is done in Table 4 below.

Table 4

Estimated Real Average Private Rates of Return on Equity 1965 - 1974

Year	Jenkins	Pesando
1965	7.97%	3.57%
1966	7.34	-10.28
1967	5.97	13.32
1968	6.85	17.53
1969	7.12	-5.29
1970	4.17	-5.01
1971	5.71	2.84
1972	6.18	21.14
1973	8.37	-8.70
1974	6.50	-34.76
Average	6.62	-0.56
Standard Deviation	1.21	16.30
Correlation Coefficient	-.10	

Source: Jenkins (1977, Table 2-5, p.35 and Pesando (1983, Table 4).

The figures from the stock market indicate that Jenkins' estimates of real private rates of return are too high. The stock market was too volatile over the period of 1965 - 1974 to really be a guide as to how much. However, if we use the data in Pesando (1983) for the longer period 1953 - 1980, then their data yields an average real private rate of return which averages 4.20%. This number is constructed as an annual average of a weighted average of the average real private rate of return on common stocks and the average real private rate of return on corporate bonds where the weights are the proportion of new investment financed from that source. This formula used is equation (1) in Appendix 1. The weights are taken from Boadway et. al. (1984a, Table A-1). Jenkins' estimates of the average real private rate of return in manufacturing average 5.94% for the period 1965 -1974 (see first column, Table 1) which is 1.74% above the Pesando number. If we adjust Jenkins' number for r_{AS} in manufacturing downward by this amount then our estimate of the social discount rate is 7.20%.

The results of this section are summarized in Table 5 below.

Table 5

A Preliminary Revision of Jenkins' Estimate of the Social Discount Rate 1965 - 1974

Jenkins' estimate of the SDR	10.02%
less:	
(i) an adjustment for the difference between the average and marginal tax rates on the industrial sector	1.86%
(ii) an adjustment for the difference between Jenkins' average private rate of return on manufacturing and rates of return observed in financial markets	0.96%
Preliminary revised estimate of the SDR	7.20%

4. Other Revisions of Jenkins' Estimate of the Social Discount Rate

Previous sections of this study have concentrated upon explaining the difference between marginal and average rates of return and on providing some estimates of these differences. This section looks at other evidence which also suggests that Jenkins overestimated the social discount rate. Moreover, this other evidence provides estimates of the social discount rate which are consistent with the revised estimate developed in section 3. There is a discussion in section 2 as to why the social discount rate r_S can be calculated as:

$$r_S = \Sigma w_i r_{MS}^i$$

where r_{MS}^i is the social opportunity cost of funding from sector i and $\Sigma w_i = 1$. The formula for the weights is $w_i = A_i\varepsilon_i/w$ where A_i is the average contribution of sector i to the financing of government expenditure and ε_i is the elasticity of funding from sector i with respect to the market rate of interest. The denominator w is the sum of the $A_i\varepsilon_i$.

Now, in Jenkins (1977), all the elasticities in this formula are assumed to equal 1 so that each weight reduces to the average contribution of that sector to government financing. However, Burgess (1981) cites three independant studies which have concluded that the interest elasticity of foreign funding to the Canadian economy ranges from 7.0 to 10.0. Accepting for the other sectors an interest elasticity of 1.0 and accepting the rates of return provided by Jenkins (see Table 3 above) Burgess then gets estimates of the social discount rate ranging from 7.86% to 8.46% for foreign debt elasticities of 10.0 and 7.0 respectively.

We have argued that the sectoral rates of return r_{MS}^i used by Jenkins are also too high. Thus we have redone Burgess' calculations using our estimate of 9.37% for the marginal social rate of return from the industrial sector, a consumption rate of interest of 3.6% rather than Jenkins' 4.11% (from Boadway et. al. (1984a, Table 1), see below) and a base rate cost of foreign capital of 4% rather than Jenkins' 6.11%. This last revision is put forward both by Burgess and by Spiro (1984). The subsequent estimates of the social discount rate are then 5.66% to 6.06% for foreign debt elasticities of 10.0 and 7.0 respectively. The details of this calculation and of Burgess' calculations are shown in Appendix 3.

Boadway et. al. (1984) have also developed estimates of the marginal social rate of return on private capital formation for the period 1963 - 1978. Their methodology is based on a multiperiod version of the model presented in Appendix 1. However, they use financial market data and estimates of tax rates to get their estimates from a version of equation (18). The weighted average of their time series observations for 1963 - 1971 is equal to 6.5%. This estimate is approximately 3.5% below the corresponding estimate in Jenkins. Part of the difference is due to the fact that Boadway et. al. neglected sales and property taxes but the main difference is that Jenkins' figures do not relate to what was happening in financial markets as we saw in the previous section.

Spiro (1987) has revised the Boadway et. al. calculations to incorporate sales and property taxes and also to take account of their imperfect measurement of tax loss offsets. His revision yields an estimate of 7.4% for the marginal rate of return to private investment and 6.6% for the social discount rate. The latter estimate is based on assuming that the interest elasticity of foreign capital inflows is equal to 7%. Our corresponding estimate of the social discount rate is equal to 6.06%.

In concluding this section it should be pointed out that all of the estimates of the social discount rate which have been discussed in this study, except Jenkins, are consistent with the conclusion of Burgess (1981) that the social discount rate is less than 7%. Burgess did not adjust his calculations for the difference between marginal and average rates of return (see Table 5). If he had done so, he probably would have arrived at an estimate of the social discount rate which was close to 6%.

5. Treatment of Risk and Uncertainty

It is customary in economics to make a distinction between risk and uncertainty. A project is risky if the known technological and economic characteristics of the project can be used to develop a model for assigning probabilities to the possible outcomes of the project. For example, planting forest land is risky in the sense that past experience provides a guide to what is the probability of having a healthy fully stocked stand in five years time. The same can be said for the possibility that a stand will be attacked by fire or pests in the future. Some tree growth simulators can also be viewed as providing probabilistic estimates of future stand dimensions. These estimates are certainly provided by Markov chain models and probably could be a byproduct of regression models such as DFSIM. DFSIM predicts regional averages of stand dimensions. It seems reasonable to suppose that deviations in actual stand growth from these averages is the sum of a large number of small independant random effects so that these deviations can be viewed as being normally distributed according to the Central Limit Theorem in statistics.

Uncertainty on the other hand refers to situations where there is no generally accepted model for estimating the probabilities of future outcomes. Over the long periods of time involved in forestry projects, there are likely to be major unforseeable changes in economic conditions and hence future wood product prices and future logging costs must be viewed as being uncertain. Procedures for making decisions in the face of uncertainty will be discussed at the end of this section. Next, however, we discuss what financial theory has to say about decision making in the face of risk.

The basic procedure that has been suggested for dealing with risk is that expected net present values (ENPV) should be used in determining which projects are worthwhile or which projects are most valuable. The expected net present value is calculated by first computing a net present value for the returns and costs associated with each possible outcome and then taking a weighted average of these NPVs where the weights are the probabilities associated with each outcome. Then, because decision makers are thought in general to be risk adverse, it is recommended that the discount rate used in these calculations be somewhat above the risk free discount rate in order that projects with positive ENPV should be able to pay a reward or compensation for the investors having undertaken the risks involved.

This procedure can be shown to be appropriate when the returns to an investment are received after a single period has elapsed. However, it is recognized now that this procedure is not appropriate for long lived investments like forestry projects (Brealey et. al. (1986, 197-9)). A simple example should serve to demonstrate what is wrong with using a risk adjusted discount rate. Suppose there is a risk that a planting will fail but after five to ten years this risk is resolved in the sense that the outcome is now known. Applying a risk adjusted discount rate to the returns from the final harvest assumes that the same degree of risk is incurred in each year from planting to harvest. This assumption is clearly false and unfairly biases decision makers against projects whose returns take a long time to be realized.

An alternative approach to valuation under risk has been put forward by Wilson (1982). He shows that the risk free rate of discount can be used in the calculation of the ENPV. He then says that a risk adjusted ENPV can be calculated by subtracting (or in some cases adding) a risk offset from the risk free ENPV. Some formulas for calculating this risk offset are provided but do not seem to be very operational. However, some general principles are also provided which do seem to be applicable to investments in silviculture.

The important question is how do silvicultural investments affect the risk faced by the people of the community. These people already face risk because they hold a portfolio of financial and other assets. It is how this risk on the total portfolio is affected by the addition of silvicultural investments to the portfolio that should determine the magnitude of the risk offset in evaluating these investments.

A simple example may serve to illustrate this point (from Lind (1982, 61)). Consider a person who has only two assets, a house and fire insurance on the house which will pay him the full value of the house if it burns down. The expected net present value of purchasing the fire insurance would have been negative as the insurance company must recover the transactions costs of providing insurance. However, buying the insurance reduces the risk on the owner's portfolio of assets to zero. The reduction in risk on the total portfolio is viewed as being preferable to holding the higher ENPV portfolio which consists of the house only. Thus it is the effect of an investment on the variability of returns from the total portfolio which is important in measuring risk.

Financial theorists therefore seem to agree that the appropriate measure of risk is the variance of returns from the project plus the covariance of returns from the project with returns from the total portfolio of the economy (i.e., the market portfolio). Since variations in returns from the total portfolio are likely to be extremly large in comparison to returns from a few projects, they go on to argue that it is the covariance term which is important in this calculation (Lind (1982, 60)).

It is also recognized in the financial literature that there are two ways in which risk can be reduced (Brealey et. al. (1986, 817)). One way is through diversification of the current portfolio. The other way is by hedging to reduce the possibilities of losses in the future. Investment on research and development for new energy technologies would be an example of risk reduction in the latter sense as it reduces the possibility of reductions in GNP which might result if there were severe increases in conventional energy prices (Lind (1982, 63)). Hedging against the loss of possible opportunities for economic growth would also reduce the risk faced by society.

Now consider the case of an investment in silviculture which will yield an increase in volume to be harvested in the future. Suppose this additional harvest is planned to take place fifty years hence. If this plan must be carried out exactly as scheduled then, since stumpage returns are based on log and lumber prices, these returns will depend on the state of demand in North American lumber markets. Lumber demand is correlated with the state of the North American economy so the silvicultural investment would increase the risk faced by society. However, suppose the forest manager is not held to the fifty years exactly but is instead given the discretion to choose the date to release the extra wood for harvest as long as the choice is reasonably near fifty years from now. Then, by using this flexibility about harvest dates the manager can hedge against the possibility of poor stumpage returns from the silviculural investment. If lumber prices are low 50 years from now, he can leave the additional volume stored on the stump. When lumber markets rebound from recession and timber industry demand for wood picks up, he will be more able to oblige as well as be able to collect better stumpage returns on this volume. The ability to respond to opportunities for good returns while conversely being able to avoid poor returns is valuable and reduces the risk faced by the provincial economy.

There are further reasons why silvicultural investments may actually reduce the riskiness of the market portfolio. First of all, these investments may contribute to avoiding or reducing falldown effects. The processing industry will then feel that they will be more able to respond to any

changes that take place in the marketplace in the future. The availability of this enlarged set of options will enhance the value of their operations. The risk faced by the industry will be reduced. Secondly, there will be less pressure on untreated forest land for wood production. Therefore these lands will be more available for alternative uses such as recreation opportunities, wilderness and aesthetics for which there is an increasing demand. As well as the higher production of these values, returns in industries such as tourism will be increased. Thirdly, forest products are an important source of export earnings for the Canadian economy. Maintaining a diversity of sources of export earnings reduces the variability of these earnings and hence reduces the social costs of adverse changes in the terms of trade facing the Canadian economy. The conclusion of these arguments then is that investments in silviculture do not need to be considered as adding to the variabilty of returns from the market portfolio and hence no risk offset needs to subtracted from the risk free ENPV in the evaluation of these projects.

Recent research in finance has actually shown ENPV calculations using the risk free discount rate may understate the net economic benefits of an investment. The understatement occurs when the investment creates options for further growth, for alternative use of assets and for expansion in the chain of value added products (Brennan and Schwartz (1985)). Since investments in silviculture have these characteristics, it is reasonable to suppose that the understatement applies to these investments. However, there is at the moment no operational method of evaluating the options created by silvicultural investments. Thus, for practical reasons, we still recommend that ENPV calculations be used to evaluate silvicultural investments.

Appendix 4 shows how to calculate ENPV when the probability of planting failure or the probability of catastrophic loss are taken into account. In each case establishment cost must be adjusted upwards and the returns from the final harvest adjusted downwards. Incorporating probablistic estimates of stand growth in these calculations would be considerably more difficult and is not recommended given our current state of knowledge of this subject. The effect of ignoring growth variability and hence using average predicted sizes in the calculation is most likely to lead to an underestimate of ENPV. This is because larger than average size trees are likely to carry a larger premium over average size trees than average sized trees carry over trees that are smaller than average size in the same proportions.

We now conclude this section with a discussion of decision making under uncertainty. Here, it should be noted that we are never completely uncertain about the future. Wood product prices have historically risen in real terms at a low rate. Manthy (1978) found for the U.S. that wood products prices had risen at a real rate of 1.4% from 1870 to 1973. Real logging costs, for timber of like quality and physical location, will undoubtedly continue to fall in the future. But the best one can do in terms of prediction is probably to identify several likely scenarios for future time patterns of returns and costs. This could involve basing estimates of the future on above average, average and below average levels in the fairly recent past and then projecting these levels to the future either in constant real terms (a safe conservative assumption) or with small real trends built in. It then takes a subjective judgement on the part of the decision maker as to whether he is willing to take the chance that the actual NPV realized by the project might turn out to be negative, or that the project selected might not turn out to yield a higher NPV than an alternative project. Finally, the literature, Baumol (1977), contains a number of criteria for making such judgements between projects which the decision maker may or may not feel comfortable with. These criteria include:

1. maximin. Choose the project whose lowest possible NPV is highest.
2. maximax. Choose the project whose highest possible NPV is highest.
3. minimax regret. Choose the project which has the least maximum regret where the maximum regret is the difference between the highest and lowest possible NPVs for the project
4. Laplace criteria. Assume each scenario is equally likely and use these probabilities to choose the project with highest possible ENPV.

All these calculations should be done using the risk free rate of discount as was argued above. In spite of the imprecise nature of these procedures, it is still better for the manager to make his subjective decisions having at hand a manageable amount of information than no information at all.

6. Recommendations

We reached the conclusion in the last section that a risk free rate of discount should be used in the cost - benefit analysis of silvicultural projects. This rate should be used in calculating an expected net present value for the project. Using expected values will tend to reduce the benefits and increase the costs of silvicultural investments as can be seen in the examples in Appendix 4. We also recommend that only stumpage payments or other revenue paid directly to the Crown be counted as benefits of these projects and only costs directly paid by the Crown be counted as costs. In theory, one should also add in any payments to factors employed in these projects which are in excess of the opportunity cost of employing those factors. However, for labour, there is no reasonable way to forecast unemployment even ten years from now so for operations that will not occur for some time there is no better assumption than that the market wage equals the opportunity cost of labour. For operations that are planned for the near future and where one objective of the project is the provision of employment where there is identifiably no alternative employment, then the wage bill for these employees might be added to the benefits of the project.

The other factor of production, capital, has probably in the past earned a rate of return above the opportunity cost of the equipment used (Percy (1986)). However, no operational method has been devised to estimate these opportunity costs. Therefore, we can only recommend that any benefits arising from this source be ignored for the present.

Several different sources, which were reviewed in sections 3 and 4, indicate that the social discount rate is between 3 and 7%. Most of the difference between these two rates must be a premium to shareholders for bearing the risk of investing in risky shares rather than in safe government bonds. We therefore feel that a discount rate of 3 - 5% can safely be used in the evaluation of silvicultural investments.

These recommendations can also be viewed as providing a test of the financial feasibity of these investments. Spiro (1984) has estimated that the real long run marginal cost of foreign borrowing for a small open economy is 4%. A positive ENPV using a discount rate of 4 or 5% implies that the project could be financed by foreign borrowing and that the stumpage payments eventually received would probably be enough to pay all government costs and to repay the principal and interest on the foreign debt.

Appendix 1

A Model for Estimating Marginal Social Rates of Return

This appendix develops a simplified macroeconomic model where private investment behaviour is assumed to take into account the Canadian tax system. It is a tax system which causes a divergence between private and social rates of return and also between average and marginal social rates of return. The model provides a theoretically consistent method of linking social marginal and average rates of returns through the operation of the tax system and in particular provides a formula for determining the former from the latter. The formula shows that marginal and average social rates of return differ if marginal and average taxation rates differ. It is the failure of Jenkins (1977) to recognize this difference that is one of the sources of error in his estimation of marginal social rates of return. Below the firm's investment problem is described. Then formulas for rates of return are developed using the assumption that the firm follows the economically efficient investment strategy.

Opportunity Cost of Capital

The model developed here will be based on the assumption that Canada is a small open economy. Thus it is assumed that Canadian capital markets are completely integrated into the international capital markets. Consequently, marginal (pre tax) rates of return on debt and equity instruments which are issued by Canadian firms are determined in international capital markets and can be viewed as being exogenous to the production - employment decisions made by these firms. The following formula then determines the real weighted average cost of capital to Canadian firms.

$$r = \beta i(1 - t^n) + (1 - \beta)\gamma - \pi \qquad (1)$$

Here i is the nominal interest rate on Canadian debt instruments, γ is the nominal rate of return on Canadian equity instruments, β is the firm's targeted debt to asset value ratio, t^n is the nominal average corporate income tax rate and π is the expected rate of inflation in Canada. The number r can be viewed as being the minimum rate of return that the firm must earn in order to ensure that its lenders and equity holders can be paid off at rates comparable to other marginal investments. Hence it is also called the opportunity cost of capital. It is a standard result in finance that r is (approximately) the discount rate the firm should use in calculating net present values for potential

investments. The assumptions made here imply that r is not affected by decisions made by the firm.

Purchase Cost of Capital

The costs of purchasing investment goods are affected in Canada by the country's tax system. In particular these costs are increased by the federal sales tax on manufactured goods and possibly by property taxes. On the other hand, these costs are reduced by the investment tax credit and the capital cost allowance. These latter advantages occur only at the end of the single period being considered here. The appropriate method of calculating the private marginal cost PMC_i of acquiring 1 unit of an investment good of type i is then given by:

$$PMC_i = C_i(1 + \varphi_i - S_i) \qquad (2)$$

where C_i is the pretax market price of the good, φ_i is the federal sales tax rate and S_i is the tax subsidy (or shield) per dollar of expenditure on this good.

The Wealth Maximizing Firm

The model of the firm presented here will assume, as is usual in economics and corporate finance theory, that the objective of the firm is to maximize the wealth of the current common shareholders. It can be shown that under certain assumptions this objective is equivalent to the objective of maximizing the net (after tax) present value of their free cash flows where the weighted average cost r of capital is used as the discount rate. The basic assumption is that the firm acts in such a way as to maintain its debt to equity ratio. Thus r is exogenous to the firm.

The firm's net (after tax) present value can be written as follows:

$$NPV = \frac{(1 - t)(PQ - wL) + VP_1}{1 + r} - \Sigma C_i K_i (1 + \varphi_i - S_i) \qquad (3)$$

Here Q is the output produced by the firm using inputs L of labour and K_i of the various types of capital . It will be assumed that the firm's production possibilities are given by a standard neoclassical production function of the form $Q = Q(K_1, \ldots, K_N, L)$. P is the market price of the output, t is the real average corporation income tax rate and VP_1 is the real market value of the firm's securities at the end of the period. Assuming perfect capital markets, VP_1 will also be equal to the market value of the firm's capital stocks at the end of the period. This proposition is called the efficient market hypothesis in financial theory.

It will now be assumed that the firm has an interior solution to its wealth maximizing problem. The optimal usage of inputs is then determined by the first order conditions:

$$PQ_L = w \tag{4}$$

$$\frac{(1 - t)PQ_i + \partial VP_1/\partial K_i}{1 + r} = C_i(1 + \varphi_i - S_i) \tag{5}$$

where $Q_L = \partial Q/\partial L$ and $Q_i = \partial Q/\partial K_i$ denote partial derivatives.

The interpretation of the first equation is the usual one that marginal value product of labour should equal the marginal cost of that labour. The interpretation of the second set of conditions is facilated by writing down an expression for VP_1 using the efficient market hypothesis. The market value of the firm's capital stocks at the end of the period will be:

$$VP_1 = \Sigma(1 + g_i)PMC_iK_i \tag{6}$$

where g_i is the gross rate of appreciation of the replacement cost of the ith type of capital adjusted for any decline in the service life of the asset and net of any physical depreciation. Thus $\partial VP_1/\partial K_i = C_i(1 + g_i)(1 + \varphi_i - S_i)$ and the conditions (5) above can be rewritten as:

$$(1 - t)PQ_i = C_i(1 + \varphi_i - S_i)\mu_i \tag{7}$$

where $\mu_i = (r - g_i)$. The term μ_i is the before tax user cost of a dollars worth of investment in capital good i. That is it is the implicit rental rate that should be paid at the end of the period for the use of the capital (Branson, 1972, 209). Equation (7) thus says that the present value of the i^{th} capital good's net of tax marginal value product should be equated to the rental rate for that good.

Finally, note that the assumption of an interior maximum requires that the average weighted cost of capital r is greater than the net rate of economic depreciation on any capital good and that the net tax subsidy per dollar of investment in any capital good be less than 1.

Private Rates of Return

Another way of interpreting (5) is that:

$$\frac{(1 - t)PQ_i + \partial VP_1/\partial K_i}{PMC_i} - 1 = r \qquad (8)$$

The left hand side of (8) is just the private marginal rate of return on investing in assets of type i which will be denoted r_{MP}^i. Thus the optimality condition says that the firm will invest in all assets up to the point where $r_{MP}^i = r$. This equation suggests that it should be possible to estimate private marginal rates of return using financial market data.

Another way of writing (3) the expression for NPV is

$$NPV = DP/(1 + r) - \Sigma C_i K_i (1 + \varphi_i) \qquad (9)$$

Here DP is the net private dividend payable to all the suppliers of finance and is given by:

$$DP = PQ - wL + VP_1 - T + (1 + r)\Sigma C_i K_i S_i \qquad (10)$$

where T is the total corporation income tax liabilities of the firm at the end of the period. The formula for T is:

$$T = t(PQ - wL) \qquad (11)$$

Now, if we use the efficient market hypothesis again then the beginning of the period market value of the firm is:

$$VP_0 = \Sigma C_i K_i (1 + \varphi_i) \qquad (12)$$

Thus (9) can be rewritten as $DP = (1 + r)VP_0 + (1 + r)NPV$ and from this expression we can see that the private average rate of return r_{AP} which is defined as $(DP/VP_0) - 1$ is greater than the weighted average cost of capital r by the average value of end of the period rents earned by firms. That is

$$r_{AP} = r + (1 + r)NPV/VP_0 \qquad (13)$$

Social Rates of Return

In considering social values we add corporate tax revenues and sales taxes net of subsidies to the private values generated by firms. Thus the social dividend on capital is

$$DS = (1 + t_s)PQ - wL + VS_1 \tag{14}$$

Here t_s is the retail sales tax rate and VS_1 is the social valuation of the capital stock at the end of the period so:

$$VS_1 = \Sigma(1 + g_i)C_iK_i \tag{15}$$

This expression differs from the expression for VP_1 in that private marginal cost $PMC_i = C_i(1 + \varphi_i - S_i)$ has been replaced by social marginal cost C_i.

The marginal social rate of return on assets of type i, $r_{MS}{}^i$ is the marginal social dividend divided by the marginal social cost of the asset all minus 1. From (14) and (15) we have $\partial DS/\partial K_i = (1 + t_s)PQ_i + (1 + g_i)C_i$. Thus:

$$r_{MS}{}^i = (1 + t_s)(PQ_i/C_i) + g_i \tag{16}$$

The economic interpretation of this condition is clear. It says that the marginal social return on investment in good i will equal the gross of tax marginal value product of the asset plus (or minus) any marginal economic depreciation. This equation is also given in words on p. 68 of Boadway et. al. (1984).

Now the marginal tax rate for the ith type of capital can be defined as $t_M{}^i = (t + t_s)PQ_i/C_i$. Equation (16) can then be further rewritten as

$$r_{MS}{}^i = (1 - t)(PQ_i/C_i) + t_M{}^i + g_i \tag{17}$$

Following Boadway et. al. (1984), the expressions for $r_{MS}{}^i$ and $t_M{}^i$ can be related to private rates of return and other parameters by using the optimal investment conditions derived above. Substituting (7) in (17) and in the expression for $t_M{}^i$ and using the formula for μ_i gives:

$$r_{MS}{}^i = (1 + \varphi_i - S_i)(r - g_i) + t_M{}^i + g_i \tag{18}$$

$$t_M{}^i = (t + t_s)(1 + \varphi_i - S_i)(r - g_i)/(1 - t) \tag{19}$$

Note that in the absense of taxes and subsidies equation (18) reduces to $r_{MS}{}^i = r$. However, in general, (18) can be rewritten as:

$$r - r_{MS}{}^i = (r - g_i)(\varphi_i - S_i) + t_M{}^i \tag{20}$$

Thus r_{MS}^i may be greater or less than r depending on the relative magnitudes of the taxation and subsidy parameters.

Next an expression is needed for the average social rate of return given by $r_{AS} = (DS/VS_0) - 1$. Using (9) and (10) to eliminate wL from DS we get:

$$DS - VS_0 = (1 + r)\Sigma(1 + \varphi_i - S_i)C_iK_i - VP_1 + (1 + r)NPV + T + t_sPQ + VS_1 - VS_0 \quad (21)$$

Dividing by VS_0, and substituting (6) for VP_1, (15) for VS_1 and ΣC_iK_i for VS_0 one then gets:

$$r_{AS} = (\Sigma(1 + \varphi_i - S_i)(r - g_i)C_iK_i)/VS_0 + (1 + r)NPV/VS_0 + t_A + (\Sigma g_iC_iK_i)/VS_0 \quad (22)$$

Here $t_A = (T + t_sPQ)/VS_0$ is the average tax rate. The last term in this expression for r_{AS} is a weighted average of the economic depreciation rates g_i where the weights $w_i = C_iK_i/\Sigma C_iK_i$ are the shares of the value of capital good i in the total value of capital. This term can be estimated using data in Jenkins (1985) and will be referred to as d_K. Following Jenkins (1977) the w_i weights will also be used to compute the economy wide marginal rate of return $r_{MS} = \Sigma r_{MS}^i w_i$ and the economy wide marginal tax rate $t_M = \Sigma t_M^i w_i$. Then from (18):

$$r_{MS} = (\Sigma(1 + \varphi_i - S_i)(r - g_i)C_iK_i)/VS_0 + t_M + d_K \quad (23)$$

From (19):

$$t_M = (t + t_s)(\Sigma(1 + \varphi_i - S_i)(r - g_i)C_iK_i)/((1 - t)VS_0) \quad (24)$$

and from (22):

$$r_{AS} = (\Sigma(1 + \varphi_i - S_i)(r - g_i)C_iK_i)/VS_0 + (1 + r)NPV/VS_0 + t_A + d_K \quad (25)$$

Estimation

The following relationship between marginal and average social rates of return is obtained by subtracting equation (25) from equation (23).

$$r_{MS} = r_{AS} - t_A + t_M - (1 + r)NPV/VS_0 \quad (26)$$

Thus Jenkins' assumption that $r_{MS} = r_{AS}$ is not warranted if either the marginal tax rate t_M is significantly below the average tax rate t_A or if economic rents NPV are substantial relative to the social value of the capital stock. Summer (1980a) claimed that monopolistic rents might

be significant in Canada. Jenkins (1980) rebutted that his data showed that the Canadian economy was quite competitive and hence NPV/VS_0 could be taken to be zero. For the purposes of estimation, we will accept Jenkins' evidence as we are not aware of any other concrete data on the size of NPV/VS_0. Equation (26) then simplifies to:

$$r_{MS} = r_{AS} - t_A + t_M \tag{26}$$

If NPV/VS_0 is not close to zero for Canada, then our estimates of r_{MS} will be too high.

We also see from equations (23) and (24) that:

$$r_{MS} = (((1 - t)/(t + t_s)) + 1)t_M + d_K$$

$$=((1 + t_s)/(t + t_s))t_M + d_K \tag{28}$$

These last two equations can then be solved for r_{MS} and t_M in terms of parameters and rates that are available from Jenkins (1977, 1985). The estimating equations for r_{MS} and t_M are:

$$r_{MS} = ((1 + t_s)(r_{AS} - t_A) - (t_s + t)d_K)/(1 - t) \tag{29}$$

$$t_M = (t + t_s)(r_{AS} - t_A - d_K)/(1 - t) \tag{30}$$

Appendix 2

Calculation of the Marginal Social Rate of Return in the Non-manufacturing Sector

Table 6

Data for the Non-manufacturing Sector 1965 - 1974

Year	Average Private Rates of Return $r_{AS} - t_A$	Net Economic Depreciation d_K	Effective Average Corporation Tax Rates t
1965	5.38%	3.72%	37.72%
1966	5.39	5.01	37.50
1967	5.79	5.76	35.61
1968	5.97	4.69	35.19
1969	5.97	5.39	35.52
1970	5.69	4.61	37.21
1971	6.36	5.10	31.45
1972	5.22	5.41	32.50
1973	6.75	4.21	29.35
1974	6.51	5.47	30.12
Average	5.90	4.94	34.22

Source: Jenkins (1977) Table 2-1, p.13; Table 2-6, p.39; Table D-1, p. 198; Table D-7, p. 212.
Jenkins (1985) Table I, p. 30.

Table 7

Estimated Real Social Rates of Return for the Non-manufacturing Sector 1965 - 1974

Year	Marginal Social Rates of Return r_{MS}	Average Social Rates of Return r_{AS}	The Difference $r_{AS} - r_{MS}$
1965	8.04%	8.79%	0.75%
1966	8.64	9.02	0.38
1967	9.06	9.32	0.26
1968	8.97	9.71	0.74
1969	9.14	9 69	0.55
1970	8.37	9.09	0.72
1971	9.23	9.79	0.56
1972	8.11	8.38	0.27
1973	9.72	10.23	0.51
1974	10.11	10.38	0.17
Average	8.94	9.44	0.50

Source: r_{MS} was calculated from (25) using the data in Table 1 and $t_s = .06$. r_{AS} is from Jenkins (1977) Table 2-7, p.43.

Appendix 3

Revised Estimates of the Social Discount Rate

Here let ε_F denote the elasticity of foreign borrowing. Suppose $r_F = f(F)$ is the supply curve for foreign funds so that r_F is the return required to attract F amount of foreign funds. Then the marginal cost σ of foreign funds is:

$$\sigma = \partial(r_F F)/\partial F = (\partial r_F/\partial F)F + r_F = r_F(1 + (1/\varepsilon_F))$$

This adjustment has been made in the table below in calculating the cost of foreign borrowing when ε_F is not 1. Burgess used 6.11% as his base rate while we use 4% from Spiro (1984).

Table 8

Estimates of Sectoral Social Opportunity Costs 1965 - 1974

Sector	Social Opportunity Costs				
	Jenkins	Burgess		Revised	
		ε_F=10	ε_F=7	ε_F=10	ε_F=7
Industrial	12.53%	12.53%	12.53	9.37	9.37
Residential Construction	7.50	7.50	7.50	7.50	7.50
Agriculture	4.48	4.48	4.48	4.48	4.48
Domestic Consumption	4.14	4.14	4.14	3.60	3.60
Foreign	6.11	6.72	6.98	4.40	4.57

The weights used in calculating the social discount rate are $wi = A_i\varepsilon_i/w$ where A_i is the average contribution of sector i to the financing of government expenditure and ε_i is the interest elasticity of funding from sector i. The denominator w is the sum of the $A_i\varepsilon_i$. The A_i used by Burgess are the weights reported by Jenkins. Jenkins assumed all the elasticities were equal to 1 while Burgess accepted this assumption for all but the foreign sector. We accept Burgess' calculations as noted in the table below.

Table 9

Estimates of Sectoral Weights 1965 - 1974

Sector	Weights				
	Jenkins	Burgess		Revised	
		ε_F=10	ε_F=7	ε_F=10	ε_F=7
Industrial	.59	.21	.27	.21	.27
Residential Construction	.16	.06	.07	.06	.07
Agriculture	.00	.00	.00	.00	.00
Domestic Consumption	.05	.02	.02	.02	.02
Foreign	.20	.71	.64	.71	.64

The social discount rate is estimated as the weighted sum of the the social opportunity costs using the weights in the last table. This procedure gives a social discount rate of 10.02% for Jenkins, 7.86% (ε_F=10) or 8.46% (ε_F=7) for Burgess and 5.66% (ε_F=10) or 6.06% (ε_F=7) for our further revision of the sectoral social opportunity costs

Appendix 4

Two Examples of Expected Net Present Values

1. Risk of Planting Failure

Suppose p is the probability of planted forest land not yielding a healthy fully stocked stand in five years time. Let C be site preparation and planting cost for the initial stand and B be the net present value of all other costs and returns associated with producing a healthy forest at age five. Using the initial planting date as the base period, then the expected net present value of growing trees on this land is:

$$ENPV = - C + (1 - p)B + p(1+r)^{-5}(- C_1 + C + ENPV)$$

Here C_1 is the cost of rehabilitating the plantation in case the original planting fails (which it is assumed is incurred immediately). This equation can then be solved for ENPV to give:

$$ENPV = [1 - p(1+r)^{-5}]^{-1}[- C + (1 - p)B + p(1+r)^{-5}(- C_1 + C)]$$

For example, if r = .05, p = .1 and C_1 = C then:

$$ENPV = - 1.085\ C + .977\ B$$

2. Risk of Catastophic Loss

Suppose p is the probability of catastrophic loss during a year assuming the stand had been unharmed in previous years. Let T be the planned harvest age, C_t be revenues net of costs associated with the planned treatments at age t and S_t be the salvage value if the stand suffers a catastrophic loss in year t. It will be assumed that the stand is salvaged and reestablished in the year following the catastrophe. The probability of a catastrophe occuring t years after stand establishment is $p(1 - p)^t$ and the probability that the stand will survive to final harvest is $(1 - p)^T$. Thus:

$$ENPV = \sum_{t=1}^{T} p(1 - p)^{t-1}(1+r)^{-t}(S_t + ENPV) + \sum_{t=0}^{T} (1 - p)^t (1+r)^{-t} C_t$$

This equation can be simplified to:

$$ENPV = \frac{(p + r)}{r + p((1-p)/(1+r))^T}\left(\sum_{1}^{T} \frac{p(1-p)^{t-1}}{(1+r)^t} S_t + \sum_{0}^{T} \frac{(1-p)^t}{(1+r)^t} C_t\right)$$

For example, if r = .05, p = .005, salvage values are zero and there is only an establishment cost C and a clearcut with net returns B at age 75, then:

$$ENPV = -1.098\ C + .754\ B/(1+r)^{75}.$$

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